MATEMATICAS. BC2 TEMA 11: Integrales inmediatas-I

1. Resolver las siguientes integrales inmediatas de **tipo potencial:**

1 $\int \frac{1}{x^2 \sqrt[5]{x^2}} dx$ 2 $\int \frac{x+1}{\sqrt[3]{x^2+2x+7}} dx$ 3 $\int sen\, 2x \cos 2x\, dx$

4 $\int \operatorname{sen}^4 x \cos x\, dx =$ 5 $\int \operatorname{tg}^2 x \sec^2 x\, dx$ 6 $\int \frac{arc\ tg\ x}{1+x^2} dx$

2. Resolver las siguientes integrales inmediatas de **tipo logarítmico:**

1 $\int \frac{2x}{1+x^2} dx$ 2 $\int \operatorname{tg} x\, dx$ 3 $\int \frac{5^{3x}}{5^{3x}+7} dx$ 4 $\int \frac{1}{x \ln x} dx$

5 $\int \frac{1}{\cos^2 x\, tgx} dx$ 6 $\int \frac{x+1}{x} dx$ 7 $\int \frac{x+1}{x-5} dx$ 8 $\int \frac{3x^3+5x}{x^2+1} dx$

3. Resolver las siguientes integrales inmediatas de **tipo exponencial:**

1 $\int e^{2x+2} dx$ 2 $\int 5^x dx$ 3 $\int 2^x 5^x dx$ 4 $\int 8^{3x+1} dx$

5 $\int \frac{e^{\ln x}}{x} dx$ 6 $\int e^{\operatorname{sen} x} \cos x\, dx$ 7 $\int \frac{e^{\operatorname{arc\ sen} x}}{\sqrt{1-x^2}} dx$

4. Resolver las siguientes integrales inmediatas de **tipo trigonométrico:**

1 $\int (3 - \operatorname{sen} x) dx$ 2 $\int sen(3x+5) dx$ 3 $\int (x+1) \operatorname{sen}(x^2+2x+3) dx$

4 $\int e^x \operatorname{sen} e^x dx$ 5 $\int \operatorname{sen} 2x\, dx$ 6 $\int sen^2\, 2x dx$ 7 $\int \operatorname{sen}^3 x\, dx$

8 $\int (2x + \cos x) dx$ 9 $\int \cos(2x+5) dx$ 10 $\int (x+1)\cos(x^2+2x+1) dx$

11 $\int \frac{\cos(\ln x)}{x} dx$ 12 $\int \cos^2 x\, dx$ 13 $\int \cos^3 3x\, dx$ 14 $\int \frac{5}{\cos^2 x} dx$

15 $\int (3 + 3\operatorname{tg}^2 x) dx$ 16 $\int \sec^2(5x+3) dx$ 17 $\int \sec^4 x\, dx$

18 $\int (3 + 3 \cot g^2 x) dx$ 19 $\int \operatorname{tg}^2 x\, dx$ 20 $\int \operatorname{sen} 3x \cos 2x\, dx$

21 $\int \frac{dx}{\operatorname{sen}^2 x \cos^2 x}$ 22 $\int \sqrt{\frac{1+x}{1-x}} dx$ 23 $\int \frac{1-\cos x}{1+\cos x} dx$

24 $\int \frac{1+sen\ x}{1-sen\ x} dx$ 25 $\int \frac{5}{x^2-4x+8} dx$ 26 $\int \frac{2x+5}{\sqrt{9-x^2}} dx$

27 $\int \frac{2^x}{\sqrt{1-4^x}} dx$ 28 $\int \frac{x}{\sqrt{9-2x^4}} dx$

SOLUCIONES

Ejercicio 1:

1 $\int \frac{1}{x^2 \sqrt[5]{x^2}} dx = \int \frac{1}{x^2 \sqrt[5]{x^2}} dx = \int x^{-2} x^{\frac{-2}{5}} dx = \int x^{\frac{-12}{5}} dx = \frac{x^{\frac{-12}{5}+1}}{\frac{-12}{5}+1} + C = = \frac{x^{\frac{-7}{5}}}{\frac{-7}{5}} + C = -\frac{5}{7\sqrt[5]{x^7}} + C$

2 $\int \frac{x+1}{\sqrt[3]{x^2+2x+7}} dx = \int \frac{x+1}{\sqrt[3]{x^2+2x+7}} dx = \frac{1}{2}\int (2x+2)(x^2+2x+7)^{\frac{-1}{3}} dx = \frac{1}{2}\frac{(x^2+2x+7)^{\frac{2}{3}}}{\frac{2}{3}} + C = \frac{3}{4}\sqrt[3]{(x^2+2x+7)^2} + C$

3 $\int sen\, 2x \cos 2x\, dx = \int sen\, 2x \cos 2x\, dx = \frac{1}{2}\int sen\, 2x \cos 2x \cdot 2\, dx = \frac{1}{4} sen^2\, 2x + C$

4 $\int \text{sen}^4 x \cos x\, dx = \int \text{sen}^4 x \cos x\, dx = \frac{1}{5}\text{sen}^5 x + C$

5 $\int \text{tg}^2 x \sec^2 x\, dx = \int \text{tg}^2 x \sec^2 x\, dx = \frac{1}{3}\text{tg}^3 x + C$

6 $\int \frac{arc\ tg\ x}{1+x^2} dx = \int \frac{arc\ tg\ x}{1+x^2} dx = \frac{1}{2}(arc\ tg\ x)^2 + C$

Ejercicio 2:

1 $\int \frac{2x}{1+x^2} dx = \int \frac{2x}{1+x^2} dx = \ln(1+x^2) + C$

2 $\int \text{tg}\, x\, dx = \int tg\, x\, dx = \int \frac{sen\, x}{\cos x} dx = -\int \frac{-sen\, x}{\cos x} dx = -\ln \cos x + C$

3 $\int \frac{5^{3x}}{5^{3x}+7} dx = \int \frac{5^{3x}}{5^{3x}+7} dx = \frac{1}{3}\frac{1}{\ln 5}\int \frac{3 \cdot 5^{3x} \ln 5}{5^{3x}+7} dx = \frac{1}{3\ln 5} \ln(5^{3x}+7) + C$

4 $\int \frac{1}{x \ln x} dx = \int \frac{1}{x \ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln(\ln x) + C$

5 $\int \frac{1}{\cos^2 x\, tgx} dx = \int \frac{1}{\cos^2 x\, tgx} dx = \int \frac{\frac{1}{\cos^2 x}}{tgx} dx = \ln(tgx) + C$

6 $\int \frac{x+1}{x} dx = \int \frac{x+1}{x} dx = \int \left(1+\frac{1}{x}\right) dx = x + \ln x + C$

7 $\int \frac{x+1}{x-5} dx = \int \frac{x+1}{x-5} dx = \int \frac{x+1-5+5}{x-5} dx = \int \frac{x-5}{x-5} dx + \int \frac{6}{x-5} dx = = x + 6\ln(x-5) + C$

8 $\int \frac{3x^3+5x}{x^2+1} dx = \int \frac{3x^3+5x}{x^2+1} dx = \int \left(3x + \frac{2x}{x^2+1}\right) dx = \frac{3}{2}x^2 + \ln(x^2+1) + C$

$3x^3 + 5x \quad | \ x^2+1$

$2x \qquad 3x$

Dividendo = divisor * Cociente + Resto

Ejercicio 3:

1 $\int e^{2x+2}\,dx = \int e^{2x+2}\,dx = \frac{1}{2}e^{2x+2} + C$

2 $\int 5^x\,dx = \int 5^x\,dx = \frac{5^x}{\ln 5}$

3 $\int 2^x\,5^x\,dx = \int 2^x\,5^x\,dx = \int 10^x\,dx = \frac{10^x}{\ln 10} + C$

4 $\int 8^{3x+1}\,dx = \int 8^{3x+1}\,dx = \frac{1}{3}\int 8^{3x+1}\;3\,dx = \frac{1}{3\ln 8}8^{3x+1} + C$

5 $\int \frac{e^{\ln x}}{x}dx = \int \frac{e^{\ln x}}{x}dx = \int \frac{1}{x}\,e^{\ln x}\,dx = e^{\ln x} + C$

6 $\int e^{\operatorname{sen} x}\cos x\,dx = \int e^{\operatorname{sen} x}\cos x\,dx = e^{\operatorname{sen} x} + C$

7 $\int \frac{e^{\operatorname{arc\,sen} x}}{\sqrt{1-x^2}}dx = \int \frac{e^{\operatorname{arc\,sen} x}}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}\,e^{\operatorname{arc\,sen} x}dx = e^{\operatorname{arc\,sen} x} + C$

Ejercicio 4:

1 $\int (3-\operatorname{sen} x)dx = \int (3-\operatorname{sen} x)dx = 3x + \cos x$

2 $\int \operatorname{sen}(3x+5)\,dx = \int \operatorname{sen}(3x+5)\,dx = \frac{1}{3}\int \operatorname{sen}(3x+5)3\,dx = -\frac{1}{3}\cos(3x+5) + C$

3 $\int (x+1)\operatorname{sen}(x^2+2x+3)dx = \int (x+1)\operatorname{sen}(x^2+2x+3)dx = \frac{1}{2}\int (2x+2)\operatorname{sen}(x^2+2x+3)dx =$

$= -\frac{1}{2}\cos(x^2+2x+3) + C$

4 $\int e^x\operatorname{sen} e^x\,dx = \int e^x\operatorname{sen} e^x\,dx = -\cos e^x$

5 $\int \operatorname{sen} 2x\,dx = \int \operatorname{sen} 2x\,dx = \frac{1}{2}\int \operatorname{sen} 2x\cdot 2\,dx = -\frac{1}{2}\cos 2x + C$

6 $\int \operatorname{sen}^2 2x\,dx = \int \operatorname{sen}^2 2x\,dx = \int \frac{1-\cos 4x}{2}dx = \frac{1}{2}x - \frac{1}{8}\operatorname{sen}4x + C$

7 $\int \operatorname{sen}^3 x\,dx = \int \operatorname{sen}^3 x\,dx = \int \operatorname{sen}^2 x\;\operatorname{sen} x\,dx = \int (1-\cos^2 x)\operatorname{sen} x\,dx =$

$= \int (\operatorname{sen} x - \cos^2 x\,\operatorname{sen} x)dx = -\cos x + \frac{1}{3}\cos^3 x + C$

8 $\int (2x+\cos x)dx = \int (2x+\cos x)dx = x^2 + \operatorname{sen} x$

9 $\int \cos(2x+5)dx = \int \cos(2x+5)dx = \frac{1}{2}\operatorname{sen}(2x+5) + C$

10 $\int (x+1)\cos(x^2+2x+1)dx = \int (x+1)\cos(x^2+2x+1)dx = \frac{1}{2}\int (2x+2)\cos(x^2+2x+1)dx =$

$= \frac{1}{2}\operatorname{sen}(x^2+2x+1) + C$

11 $\int \frac{\cos(\ln x)}{x}dx = \int \frac{\cos(\ln x)}{x}dx = \int \cos(\ln x)\frac{1}{x}dx = \text{sen}(\ln x) + C$

12 $\int \cos^2 x\, dx = \int \cos^2 x\, dx = \int \left(\sqrt{\frac{1+\cos 2x}{2}}\right)^2 dx = \int \frac{1+\cos 2x}{2}dx =$

$\frac{1}{2}\int (1+\cos 2x)dx = \frac{1}{2}\left(x+\frac{1}{2}\text{sen}\,2x\right) + C = \frac{1}{2}x + \frac{1}{4}\text{sen}\,2x + C$

13 $\int \cos^3 3x\, dx = \int \cos^3 3x\, dx = \int \cos^2 3x \cos 3x\, dx = \int (1-sen^2 3x)\cos 3x\, dx =$

$= \int \cos 3x\, dx - \int sen^2 3x \cos 3x\, dx = \frac{1}{3}sen\, 3x - \frac{1}{9}sen^3 3x + C$

14 $\int \frac{5}{\cos^2 x}dx = \int \frac{5}{\cos^2 x}dx = 5\,tgx + C$

15 $\int (3+3\text{tg}^2 x)dx = \int (3+3tg^2 x)dx = 3\int (1+tg^2 x)dx = 3tg\, x + C$

16 $\int \sec^2(5x+3)dx = \int \sec^2(5x+3)dx = \frac{1}{5}\int \sec^2(5x+3)5dx = \frac{1}{5}tg\,(5x+3) + C$

17 $\int \sec^4 x\, dx = \int \sec^4 x\, dx = \int \sec^2 x\, \sec^2 x\, dx = \int (1+tg^2 x)\sec^2 x\, dx =$

$= \int (\sec^2 x + \sec^2 x\; tg^2 x)dx = tg\, x + \frac{1}{3}tg^3 x + C$

18 $\int (3+3\cot g^2 x)dx = \int (3+3\cot g^2 x)dx = 3\int (1+\cot g^2 x)dx = -3\cot g\, x + C$

19 $\int \text{tg}^2 x\, dx = \int \text{tg}^2 x\, dx = \int (1+\text{tg}^2 x - 1)dx = \int (1+\text{tg}^2 x)dx - \int dx = \text{tg}\,x - x + C$

20 $\int \text{sen}\,3x \cos 2x\, dx = \int \text{sen}\,3x\cos 2x\, dx = \frac{1}{2}\int 2\,\text{sen}\,3x\cos 2x\, dx = = \frac{1}{2}\int (\text{sen}\,5x + \text{sen}\,x)dx = \frac{1}{2}\left(-\frac{\cos 5x}{5} - \cos x\right) + C$

$\text{sen}\,A + \text{sen}\,B = 2\,\text{sen}\frac{A+B}{2}\cos\frac{A-B}{2}$ $\quad \begin{cases} \frac{A+B}{2} = 3x \\ \frac{A-B}{2} = 2x \end{cases}$ $\quad A = 5x \quad B = x$

21 $\int \frac{dx}{\text{sen}^2 x\cos^2 x} = \int \frac{dx}{\text{sen}^2 x\cos^2 x} = \int \frac{\text{sen}^2 x + \cos^2 x}{\text{sen}^2 x\cos^2 x}dx =$

$\int \frac{\text{sen}^2 x}{\text{sen}^2 x\cos^2 x}dx + \int \frac{\cos^2 x}{\text{sen}^2 x\cos^2 x}dx = = \int \frac{dx}{\cos^2 x} + \int \frac{dx}{\text{sen}^2 x} = \text{tg}\,x - \text{cotg}\,x + C$

22 $\int \sqrt{\frac{1+x}{1-x}}dx = \int \sqrt{\frac{1+x}{1-x}}dx = \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx =$

$= \int \frac{1}{\sqrt{1-x^2}}dx - \frac{1}{2}\int (-2x)(1-x^2)^{\frac{-1}{2}}dx =$

$= arc\; sen\, x - \sqrt{1-x^2} + C$

23 $\int \frac{1-\cos x}{1+\cos x}dx = \int \frac{1-\cos x}{1+\cos x}dx = \int \frac{(1-\cos x)^2}{1-\cos^2 x}dx = \int \frac{1-2\cos x+\cos^2 x}{sen^2 x}dx =$

$$= \int \frac{1}{sen^2 x}dx - 2\int \cos x\ sen^{-2}x\,dx + \int \cot g^2\, x\,dx =$$

$$= \int \frac{1}{sen^2 x}dx - 2\int \cos x\ sen^{-2}x\,dx + \int \left[\left(1+\cot g^2\, x\right)-1\right]dx =$$

$$= -\cot g\, x + \frac{2}{sen\, x} - \cot g\, x - x\ + C = = -2\cot g\, x + \frac{2}{sen\, x}\ - x + C$$

24 $\int \frac{1+sen\ x}{1-sen\ x}dx = \int \frac{(1+sen\ x)^2}{1-sen^2 x}dx = \int \frac{1+2sen\, x+sen^2 x}{\cos^2 x}dx =$

$$= \int \frac{1}{\cos^2 x}dx + 2\int \frac{senx}{\cos^2 x}dx + \int \frac{sen^2 x}{\cos^2 x}dx =$$

$$= \int \frac{1}{\cos^2 x}dx - 2\int (-senx)\cos^{-2} x\,dx + \int tg^2\, x\,dx =$$

$$= \int \frac{1}{\cos^2 x}dx - 2\int (-senx)\cos^{-2} x\,dx + \int\left[\left(1+tg^2\, x\right)-1\right]dx =$$

$$= tg\, x + \frac{2}{\cos x} + tg\, x - x\ + C = = 2tg\, x + \frac{2}{\cos x} - x\ + C$$

25 $\int \frac{5}{x^2-4x+8}dx = \int \frac{5}{x^2-4x+8}dx = \int \frac{5}{x^2-4x+4+4}dx = \int \frac{5}{4+(x-2)^2}dx =$

$$= \frac{5}{4}\int \frac{dx}{1+\left(\frac{x-2}{2}\right)^2} = \frac{5}{4}\cdot 2\int \frac{\frac{1}{2}}{1+\left(\frac{x-2}{2}\right)^2}dx = = \frac{5}{2}arc\ tg\left(\frac{x-2}{2}\right)\ + C$$

26 $\int \frac{2x+5}{\sqrt{9-x^2}}dx = \int \frac{2x}{\sqrt{9-x^2}}dx + \int \frac{5}{\sqrt{9-x^2}}dx = -\int (9-x^2)^{-\frac{1}{2}}(-2x)\,dx + \frac{5}{3}\cdot 3\int \frac{\frac{1}{3}}{\sqrt{1-\left(\frac{x}{3}\right)^2}}dx =$

$$= -\frac{(9-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + 5\,\text{arc sen}\left(\frac{x}{3}\right) + \text{C} = -2\sqrt{9-x^2} + 5\,\text{arc sen}\left(\frac{x}{3}\right) + \text{C}$$

27 $\int \frac{2^x}{\sqrt{1-4^x}}dx = \int \frac{2^x}{\sqrt{1-(2^x)^2}}dx = \frac{1}{\ln 2}\int \frac{2^x \ln 2}{\sqrt{1-(2^x)^2}}dx = = \frac{1}{\ln 2}\text{arc sen}(2^x)\ + \text{C}$

28 $\int \frac{x}{\sqrt{9-2x^4}}dx = \int \frac{x}{\sqrt{9\left[1-\left(\frac{\sqrt{2}\,x^2}{3}\right)^2\right]}}dx = \frac{1}{3}\frac{3}{2\sqrt{2}}\int \frac{\frac{2\sqrt{2}}{3}x}{\sqrt{1-\left(\frac{\sqrt{2}\,x^2}{3}\right)^2}}dx = \frac{1}{2\sqrt{2}}\text{arc sen}\left(\frac{\sqrt{2}}{3}x^2\right) + \text{C}$