(Continuación)

$c)\quad \lim_{x\to-\infty}\left(x^2+3x^3+1\right)=\boxed{-\infty}$ (porque el término de mayor grado es negativo)

$$d)\quad \lim_{x\to0}\frac{3x^4}{x^3+x^2}\underbrace{=}_{IND\,\frac{0}{0}}\lim_{x\to0}\frac{3x^2\cdot x^2}{x^2\left(x+1\right)}=\lim_{x\to0}\frac{3x^2}{\left(x+1\right)}=\frac{0}{0+1}=\boxed{0}$$

$$e)\quad \lim_{x\to1}\frac{x^3-1}{x^2-1}\underbrace{=}_{IND\,\frac{0}{0}}\lim_{x\to1}\frac{\left(x-1\right)\left(x^2+x+1\right)}{\left(x+1\right)\left(x-1\right)}=\lim_{x\to1}\frac{x^2+x+1}{x+1}=\boxed{\frac{3}{2}}$$

$$f)\quad \lim_{x\to5}\frac{x^2-25}{x^2-5x}\underbrace{=}_{ÎND\,\frac{0}{0}}\lim_{x\to5}\frac{\left(x+5\right)\left(x-5\right)}{x\left(x-5\right)}=\lim_{x\to5}\frac{x+5}{x}=\boxed{2}$$

$$g)\quad \lim_{x\to\sqrt{5}}\frac{x-\sqrt{5}}{x^2-5}\underbrace{=}_{IND\,\frac{0}{0}}\lim_{x\to\sqrt{5}}\frac{x-\sqrt{5}}{\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)}=\lim_{x\to\sqrt{5}}\frac{1}{x+\sqrt{5}}=\boxed{\frac{1}{2\sqrt{5}}}$$

$$h)\quad \lim_{x\to1}\frac{\sqrt{x+3}-2}{x-1}\underbrace{=}_{IND\,\frac{0}{0}}\lim_{x\to1}\frac{\left(\sqrt{x+3}-2\right)\cdot\left(\sqrt{x+3}+2\right)}{\left(x-1\right)\left(\sqrt{x+3}+2\right)}=\lim_{x\to1}\frac{x+3-4}{\left(x-1\right)\left(\sqrt{x+3}+2\right)}=$$

$$=\lim_{x\to1}\frac{x-1}{\left(x-1\right)\left(\sqrt{x+3}+2\right)}=\lim_{x\to1}\frac{1}{\sqrt{x+3}+2}=\frac{1}{\sqrt{4}+2}=\boxed{\frac{1}{4}}$$

$$i)\quad \lim_{x\to+\infty}\left(\frac{4x+1}{2x}\right)^x=2^\infty=\boxed{\infty}$$

Ejercicio 9

Calcula los siguientes límites:

$a)\quad \lim_{x\to 3}\frac{x+1}{x-1}=\frac{4}{2}=\boxed{2}$ No hay indeterminación

$$b)\quad \lim_{x\to\infty}\frac{x+1}{x-1}\underbrace{=}_{IND\ \frac{\infty}{\infty}}\lim_{x\to\infty}\frac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}-\frac{1}{x}}=\lim_{x\to\infty}\frac{1+\frac{1}{x}}{1-\frac{1}{x}}=\frac{1+0}{1-0}=\boxed{1}$$

$$c)\quad \lim_{x\to 1}\frac{x^2-1}{x-1}\underbrace{=}_{IND\ \frac{0}{0}}\lim_{x\to 1}\frac{(x+1)(x-1)}{x-1}=\lim_{x\to 1}(x+1)=\boxed{2}$$

$$d)\quad \lim_{x\to 0}\frac{x}{1-\sqrt{1-x}}\underbrace{=}_{IND\ \frac{0}{0}}\lim_{x\to 0}\frac{x\left(1+\sqrt{1-x}\right)}{\left(1-\sqrt{1-x}\right)\left(1+\sqrt{1-x}\right)}=\lim_{x\to 0}\frac{x\left(1+\sqrt{1-x}\right)}{1-(1-x)}=$$

$$=\lim_{x\to 0}\frac{x\left(1+\sqrt{1-x}\right)}{x}=\lim_{x\to 0}\left(1+\sqrt{1-x}\right)=\boxed{2}$$

$$e)\quad \lim_{x\to\infty}\frac{\sqrt{x^2+x}}{x}\underbrace{=}_{IND\ \frac{\infty}{\infty}}\lim_{x\to\infty}\frac{\sqrt{\frac{x^2+x}{x^2}}}{\frac{x}{x}}=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x}}}{1}=\sqrt{1+0}=\boxed{1}$$

$$f)\quad \lim_{x\to 3}\frac{\sqrt{x+6}-3}{x-3}\underbrace{=}_{IND\ \frac{0}{0}}\lim_{x\to 3}\frac{\left(\sqrt{x+6}-3\right)\left(\sqrt{x+6}+3\right)}{(x-3)\left(\sqrt{x+6}+3\right)}=\lim_{x\to 3}\frac{x+6-9}{(x-3)\left(\sqrt{x+6}+3\right)}=$$

$$=\lim_{x\to 3}\frac{x-3}{(x-3)\left(\sqrt{x+6}+3\right)}=\lim_{x\to 3}\frac{1}{\sqrt{x+6}+3}=\frac{1}{\sqrt{9}+3}=\boxed{\frac{1}{6}}$$